Exploring Nonlinear Dynamics of Iris Deformation

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Outline

• Related Work
• Physiology of the Iris Muscles
• Overview of Continuum Mechanics
• Mathematical Formulation
• Further Mathematical Analysis & Numerical Set-Up
• Numerical Results
• Final Thoughts
Related Work

• Current iris recognition algorithms do not fully take into account the deformation of the iris tissue.
  – At most assume a simple transformation

• Recent iris recognition research has found that pupil dilation has an effect on iris recognition performance.
  – Ma. et. al (2004) quantified the number of false non matches due to pupil dilation
  – Thornton et. al (2007) – Used Bayesian estimation to consider the level of iris deformation.
  – Wei et. al (2007) – Modeled the nonlinear iris stretch as a sum of a linear and a Gaussian deviation.

• Past approaches used pattern recognition
  – Pattern recognition can be helpful, but limitations exist because they are dependent on a particular dataset.
Related Work (continued)

- Pupil dilation has been explored in other fields ("biologically-related" and computer graphics (CG))
  - Moon & Spencer (1944) – first model to describe pupil dynamics from empirical data.
  - Ellis (1981) – empirically determined the latency as well as constriction and dilation velocities.
  - Stark (1984) – expressed the pupil dynamics as a feedback control system.
  - Pamplona (2008) – developed a concrete version of Longtin’s model. [CG]

- However, there has been limited attention to exploring iris dynamics
  - Rohen (1951) – the first researcher to study the collagen form of the iris
  - Wyatt (2000) – used Rohen’s formulation to model the iris deformation as a sum of a linear stretch and nonlinear deviation
  - Wei et. al (2007) – used pattern recognition techniques to determine that the nonlinear stretch is Gaussian.
  - Pamplona (2008) – used an image-based model for iridal deformation. [CG]

- Therefore, there’s a need to build a model that takes into account the physiology of the iris!
Physiology of the Iris Muscles

- Sphincter muscle
- Radial muscle

Pupil
Overview of Continuum Mechanics

- **Displacement**: Vector quantity that describes how far a piece of material has moved. Denoted by the function \( u \).

- **Strain**: Tensor (vector) measuring how much in each direction the material is stretching or compressing at some spot. Denoted by \( \varepsilon \).

- **Stress**: Tensor (vector) that describes the force per unit area on a particle.

- **Equilibrium Condition**: Sum of all the forces on the particle = \( ma \).
Overview of Continuum Mechanics
(continued)

- **Infinitesimal (Small) deformation** – deformation that occurs such that the original and deformed states are insignificant.
  - This is common in hard tissues and materials as well as fluids

- **Finite (Large) deformation** – deformation that occurs such that the original and deformed states are significantly different from each other.
  - This is common in soft tissues and material

- **Anisotropic material** – material that has varying material parameters in many different directions (i.e., directionally dependent material)

- **Orthotropic material** – material that has varying material parameters only in the orthogonal directions.

- **Isotropic material** – material is uniform (the same) in all directions.
Problem Statement

- If the pupil dilates from some initial radius $R_1$ to some final radius $R_{\text{final}}$, what is the deformation (displacement) at every point inside the iris region? That is, what is $u(r,\theta)$?
Mathematical Formulation

Start with the original strain-displacement relationship in the Cartesian coordinate system (sum over repeated indices):

\[
\mathcal{E}_{ij} = \frac{1}{2} \left\{ \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j} \right\}
\]
Mathematical Formulation (continued)

Convert this relationship to polar coordinates

Strain along radial direction

\[ \varepsilon_{rr} = \frac{\partial u_r}{\partial r} \frac{1}{2} \left( \left( \frac{\partial u_r}{\partial r} \right)^2 \right) \]

Strain along angular direction

\[ \varepsilon_{\theta\theta} = \frac{u_r}{r} \frac{1}{2r^2} \left( \frac{u_r}{r} \right)^2 \]
Mathematical Formulation & Analysis (continued)

Similarly for the equilibrium condition we have

\[ \frac{\partial \sigma_r}{\partial \theta} \cdot \frac{\sigma_r - \sigma_\theta}{r} = 0 \]

Shear Stress

Normal stress along radial and angular direction
Mathematical Formulation (continued)

Therefore, we have the following Boundary Value Problem (BVP)

\[ u'' + \frac{u'}{r} - \frac{\zeta u}{r^2} - \frac{(1-\nu\zeta)}{2r} (u')^2 - \frac{(\nu-1)\zeta}{2r} \left( \frac{u}{r} \right)^2 - \frac{1}{2} \frac{d}{dr} (u')^2 - \frac{\nu\zeta}{2} \frac{d}{dr} \left( \frac{u}{r} \right)^2 = 0 \]

\[ u(r_1) = \mu_1 \quad \text{Inner iris boundary moves} \]
\[ u(r_2) = 0 \quad \text{Outer iris boundary is stationary} \]
\[ \zeta = \frac{E_\theta}{E_r} \quad \text{Ratio of Young’s Moduli along angular and radial directions} \]
Further Mathematical Analysis & Numerical Set-Up

- **Additional Mathematical Analysis**
  - Additional mathematical analysis was done in order to relate the iris dynamics to the mathematical theory
    - **Existence** was preserved due to the fact that our equations were taken from the equations of solid mechanics.
    - **Uniqueness** was determined by using our knowledge of nonlinear operators
    - **Perturbation analysis** was used to determine and predict the nonlinear behavior.

- **Numerical Set Up**
  - We performed a numerical simulation of our model and compared it to Pamplona’s results for the various range of pupil dilation.
    - We also adopted Pamplona’s metrics to numerically analyze the nonlinear effects
      - Distance from interior point (or the pupil center) of the annular region
      - Distance between the tracked interior iris point to the inner radius (or pupil border)
      - (tracked interior point to pupil border)/(width of the iridal disk)
Numerical Results

Simulation Results for 0.5mm Dilation (Orthotropic Material)
Solution: $u(r)$

Simulation Results for 0.5mm Dilation (Isotropic Material)
Solution: $u(r)$
Numerical Results

Simulation Results for 1.5mm Dilation (Orthotropic Material)

Solution: $u(t)$

Final Positions: $u(t)+\Phi$

Simulation Results for 1.5mm Dilation (Isotropic Material)

Solution: $u(t)$

Final Positions: $u(t)+\Phi$
Numerical Results

Simulation Results for 3mm Dilation (Orthotropic Material)

Solution: $u(t)$

Final Positions: $u(t) + r$

Simulation Results for 3mm Dilation (Isotropic Material)

Solution: $u(t)$

Final Positions: $u(t) + r$

Linear Solution Visualization

Nonlinear ODE Solution Visualization
Numerical Results

Invariance Results for Orthotropic Material

Invariance (Percent in Iris) vs Pupil Diameter (mm)

Distance from Pupil Border vs Pupil Diameter (mm)

Invariance Results for Isotropic Material

Invariance (Percent in Iris) vs Pupil Diameter (mm)

Distance from Pupil Border vs Pupil Diameter (mm)

Distance from Pupil Center vs Pupil Diameter (mm)
Final Thoughts

• Conclusions
  – We were able to use our knowledge of biomechanics to create a mathematical model to describe iris dynamics
  – We related the iris physiology to mathematical theory in our analysis
  – Showed the similarity in behavior between both the orthotropic and isotropic case.

• Future Work
  – Study the validity of our model by collecting iris images exhibiting various states of dilation
  – Design a new iris normalization scheme based on the proposed model
  – Explore the effects of dilation on the 3D structure of the iris.

• Submissions
  – Clark et. al “Exploring 2D Nonlinear Dynamics in Iris Deformation”
    • Submitted and Under Review in the SIAM Journal of Applied Mathematics (SIAP)
  – Clark et. al “A Theoretical Model for Describing Iris Dynamics”
    • To appear as a book chapter in “Handbook of Iris Recognition” (Editors Bowyer and Burge)
Iris Normalization

- Daugman’s rubber sheet model is typically used to account for variations in iris size caused by:
  - pupil dilation due to changes in ambient lighting condition
  - distance from the camera during image acquisition

Cartesian Coordinates  Pseudo-Polar Coordinates