

# Identification Model with Independent Matching Scores

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## 1. Introduction

Biometric applications can be divided into two types: verification tasks and identification tasks. For verification tasks a single matching score is given, and application accepts or rejects a matching attempt by thresholding the matching score. Based on the threshold value a performance characteristics of FAR and FRR can be estimated. For identification tasks a set of  $N$  matching scores  $\{s_1, s_2, \dots, s_N\}, s_1 > s_2 > \dots > s_N$ , is produced for  $N$  enrolled persons. The person can be identified as an enrollee corresponding to the best ranked score  $s_1$ , or the identification attempt can be rejected. Based on the decision algorithm FAR and FRR can be estimated.

The usual decision algorithm for identification involves setting some threshold  $\theta$  and accepting top ranked score  $s_1$  if this score is greater than threshold:  $s_1 > \theta$  [1, 3]. But, as it is frequently noted in general pattern classification tasks, it is advantageous to use other scores in addition to the best score to make a decision about accepting classification results [2]. In this work we define decision rule based on two best ranked scores, and show its superiority over decision rule using single best ranked score.

## 2. Identification Model Framework

Let  $\{s_1, s_2, \dots, s_N\}, s_1 > s_2 > \dots > s_N$  be the set of matching scores (note that indexes correspond to score rank, and not to the enrolled person). The matching event can be one of two classes:  $s_1$  is the genuine match score, and  $s_1$  is the impostor match score. Denote corresponding probability density functions of these two classes as  $p_{gen}(s_1, \dots, s_N) = p(s_1, \dots, s_N | s_1 \text{ is genuine score})$  and  $p_{imp}(s_1, \dots, s_N) = p(s_1, \dots, s_N | s_1 \text{ is impostor score})$ . If we were able to reliably estimate these densities, then we could use Bayes decision theory to optimally separate good (best ranked score is the genuine score) from bad (best ranked score is the impostor score) identification attempts.

But, since the number of enrolled persons  $N$  is usually big, it is not feasible to estimate these densities. Instead we can restrict ourselves to considering first two scores and estimating probability density functions in two-dimensional score space:  $p_{gen}(s_1, s_2) = p(s_1, s_2 | s_1 \text{ is genuine score})$  and  $p_{imp}(s_1, s_2) = p(s_1, s_2 | s_1 \text{ is impostor score})$ . Bayes decision theory holds that optimal decision surfaces are defined by the likelihood ratio:  $L(s_1, s_2) = \frac{p_{gen}(s_1, s_2)}{p_{imp}(s_1, s_2)}$ . Thus instead of original decision rule of accepting match attempt if  $s_1 > \theta$  we use new decision rule: approximate  $L(s_1, s_2)$  and accept match attempt if  $L(s_1, s_2) > \theta$ .

## 3 Artificial Example

Let  $p_g$  and  $p_i$  denote the densities of genuine and impostor matching scores. We fix the densities for our example as  $p_g(s) = c_g e^{-\frac{(1-s)^2}{2\sigma_g^2}}$  and  $p_i(s) = c_i e^{-\frac{s^2}{2\sigma_i^2}}$  where  $s \in [0, 1]$ ,  $\sigma_g = .3$ ,  $\sigma_i = .2$  and  $c_g$  and  $c_i$  are normalizing constants.

The main assumption which we make in this example is that the scores  $s_i$  produced during matching input pattern against all classes are independent random variables. One score from the truth class is sampled from  $p_g$  and remaining  $N - 1$  scores are sampled from  $p_i$ . This assumption is rather restrictive and generally it is not true; we will discuss it later. Using independence assumptions we are able to calculate the joint density of best and second-best scores for two classes: a class where best score comes from genuine match and a class where best score comes from impostor match.

$$\begin{aligned} p_{gen}(s_1, s_2) &= p_g(s_1) p_i(s_2) F_{N-2}^i(s_2) (N-1) \\ p_{imp}(s_1, s_2) &= p_i(s_1) p_i(s_2) F_{N-3}^i(s_2) (N-2)(N-1) F_1^g(s_2) \\ &\quad + p_i(s_1) * p_g(s_2) * F_{N-2}^i(s_2) * (N-1) \end{aligned} \quad (1)$$

In above formulas  $F_n^g(s)$  or  $F_n^i(s)$  denote probabilities of  $n$  genuine or impostor scores be less than  $s$  :  $F_n^g(s) = (\int_0^s p_g(t) dt)^n$ ,  $F_n^i(s) = (\int_0^s p_i(t) dt)^n$ . Note that the probability of impostor match having best score,  $p_{imp}(s_1, s_2)$ , is the sum of probabilities of two events: second match is the impostor match, and second match is the genuine match.

Figures show ROC curves for acceptance decision based on single  $s_1$  score - original thresholding and acceptance decision based on  $s_1$  and  $s_2$  scores. Note that the benefit of employing second best score  $s_2$  is getting smaller as the number of enrolled persons increases. This can be explained by the property that cumulative distribution function for the best impostor score approaches step function ([1] has a picture illustrating this property), so thresholding near step position has greater influence than second best score.

#### 4. Dependent Scores

The main assumption used in our example is the independence of matching scores. But this is rarely a case in real life matchers. For example, frequently matching score includes some measure of input signal quality. Since the quality of input is the same for all matching attempts, we expect that scores  $s_1, s_2, \dots, s_N$  will be dependent. Is it still beneficial to use second-best score for making identification decision if scores are dependent?

Most of the times we have to answer 'yes' to this question. In [2] we gave two extreme examples of using second ranked score. Note that in ideal situation second ranked score serves as an additional feature, and adding one feature can not (ideally) worsen classification. Thus the worst scenario is where no improvement can be gained by using second ranked score. The best scenario is where using second ranked score allows complete separation of good (genuine score is best) from bad (impostor score is best) identification attempts. Both scenarios are possible for dependent scores, and the case of independent scores has average improvement.

#### 5. Conclusion

The purpose of artificial example and independence condition on matching scores was to reveal that the benefit from using a combination of scores, instead of only one best score, comes naturally. The improvement is explained by explicitly stating that we deal with identification process - the true class is one among  $N$  matched classes. In case of dependent scores total improvement of using score combination can be considered as composite of two parts: improvement due to identification process assumption and improvement due to score dependency.

#### References

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